Phenomenological model for symmetry breaking in a chaotic system

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We assume that the energy spectrum of a chaotic system undergoing symmetry-breaking transitions can be represented as a superposition of independent level sequences, one increasing at the expense of the others. The relation between the fractional level densities of the sequences and the symmetry-breaking interaction is deduced by comparing the asymptotic expression of the level-number variance with the corresponding expression obtained using the perturbation theory. This relation is supported by a comparison with previous numerical calculations. The predictions of the model for the nearest-neighbor-spacing distribution and the spectral rigidity are in agreement with the results of an acoustic resonance experiment.

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I. INTRODUCTION

Random matrix theory provides a framework for describing the statistical properties of spectra for quantum systems whose classical counterpart is chaotic [1,2]. It models the Hamiltonian of a chaotic system by an ensemble of N-dimensional random matrices, subject to some general symmetry constraints. Time-reversal-invariant quantum system having integral spins are represented by a Gaussian orthogonal ensemble (GOE) of random matrices while those having half-integral spins are modeled by a Gaussian symplectic ensemble (GSE). Chaotic systems without time-reversal invariance are represented by the Gaussian unitary ensemble (GUE). Symmetries associated with quantum numbers always involve a structure of the Hamiltonian matrices, which is reflected in the composition of the energy spectra. When the system has such a symmetry with M eigenvalues, the Hamiltonian of the system is block diagonal. Each block represents an eigenvalue (or a set of eigenvalues) of the symmetry operator, and may be considered as a member of one of the above-mentioned canonical ensembles. The energy spectrum is given by a superposition M of independent sequences, each one representing one of the Hamiltonian blocks. Modifications accounting for the symmetry breaking are realized by introducing coupling blocks belonging to different quantum numbers. The first attempt in this direction is due to Rosenzweig and Porter [3]. Their model was expanded by French *et al.* [4], who applied the perturbation theory to study transitions between different classes of symmetry. Guhr and Weidenmüller [5] applied this model to study mixing of states with isospin $T=0$, 1 in nuclei. Leitner *et al.* [6,7] obtained closed-form expressions for the nearest-neighbor-spacing (NNS) distributions within its framework by applying the perturbation theory. Their results have been applied to experimental spectra for two coupled microwave billiards [8] by Barbosa and Harney [9]. One, however, may have difficulties with this model when the number of the symmetry eigenvalues M is more than two. More information has to be put in the theory since the influence of symmetry breaking on different eigenfunctions may be different. Moreover, one may have difficulties in applying Leitner's perturbation formulas for symmetries with large M, which are valid for energy intervals of length considerably exceeding M. Numerical investigations for breaking such symmetries require diagonalizing larger matrices, which are built of sizable blocks, in order to achieve a statistical significance comparable to that of modern experiments. Additionally, the number of allowable symmetry eigenvalues is not always fixed, as in the case of multiple collective excitation of nuclei. For example, the maximum number of phonons realized in the nuclear vibrational model is subject to dispute [11].

In the present paper, we consider a simple model for gradual symmetry breaking in a chaotic system. This model represents the energy spectrum as independent level sequences not only in the absence of the symmetry breaking perturbation, but also during the whole transition until the symmetry is totally violated. During the crossover from full to no symmetry, one of the sequences grows at the expense of the other sequences until it totally exhausts the spectrum. The proposed model leads to approximate expressions for the nearest-neighbor and next-to-nearest-neighbor spacing distributions [10] which depend on a single parameter, namely the mean level density of the sequences. These have been successfully applied to the analysis of the spectra of coupled microwave resonators [8], which are not described by a Hamiltonian divided into a symmetry-breaking term and a perturbation. The model provides a satisfactory description for level statistics of low-lying 2^+ states of even-even nuclei [12] without explicit knowledge of their symmetry properties. This paper presents additional arguments in favor of that model. Section II shows that the energy spectrum obtained in the Rosenzweig-Porter model [3] may approximately be represented as a composite spectrum of level sequences. Section III uses the asymptotic values for the expressions of level-number variance obtained by French *et al.* [4] for systems undergoing a symmetry breaking to find a relation between the fractional level density of the sequences and the symmetry-breaking perturbation strength. This relation is tested in Sec. IV by a comparison of the NNS distributions of the composite spectrum with the ones obtained by Leitner [6] through a diagonalization of the Rosenzweig-Porter Hamiltonian. Section V shows that the model under investigation is consistent with the results of the acoustic-resonance experiment by Andersen *et al.* [13]. The conclusion and summary of this work are given in Sec. VI.

II. A MODEL FOR SYMMETRY BREAKING

The purpose of this section is to show that the level spectrum of a chaotic system with a conserved symmetry can still be expressed as a superposition of independent sequences, even when the symmetry is partially violated. For concreteness, we restrict consideration in this section to the transition from two independent GOE spectra to a single GOE spectrum. The extension to other universality classes is trivial. We assume that the system is described by the Rosenzweig-Porter model [3]. The Hamiltonian of the perturbed system can be written as a sum of a block-diagonal matrix, representing the case when the symmetry is conserved, and a perturbation responsible for symmetry breaking. When the symmetry has two eigenvalues of equal degeneracy, the Hamiltonian takes the form

$$H = \begin{pmatrix} H_1 & 0 \\ 0 & H_2 \end{pmatrix} + \varepsilon \begin{pmatrix} 0 & V \\ V^{\dagger} & 0 \end{pmatrix}, \tag{1}$$

where H_1 and H_2 are GOE matrices with elements having a variance $v^2(1+\delta_{ij})$, and V is a random matrix with elements having the same variance so that $\varepsilon=1$ causes H as a whole to be a GOE matrix.

Because the system is chaotic, the eigenvalues of the symmetry are expected to spread out over phase space. Thus, all of the matrix elements that couple different eigenstates are statistically equivalent. Keeping this in mind, we consider a simple version of the Rosenzweig-Porter model in which $H_1=H_2$, hoping that the obtained results approximately apply for independent diagonal Hamiltonian blocks. We start with the case in which the symmetry is fully conserved ($\varepsilon=0$). The total wave function is given by a product of an eigenfunction of the symmetry operator and an eigenfunction that depends on all coordinates except the one that represents the symmetry. For concreteness, we shall refer to these as the spatial coordinates, although other variables may be involved. In the case under consideration, the symmetry operator S has two eigenvalues, say s_1 and s_2. It can be represented by the 2×2 matrix, $\mathrm{diag}(s_1,s_2)$, with corresponding eigenvectors α_1 and α_2. Wave functions of a chaotic system have nearly a uniform phase-space distribution. Almost nothing happens to the spatial components of the wave functions upon introducing the symmetry-breaking perturbation. In this case, the effect of symmetry breaking can approximately be taken into account by simply introducing a new symmetry operator S_δ that contains additional nondiagonal elements, each equal to δ, which is proportional to ε. The wave function of each perturbed state is given by a spatial wave function that does not depend on the symmetry breaking, multiplied by one of the two eigenfunctions $\beta_{1,2}$ of S_δ having eigenvalues $\lambda_{1,2}=\frac{1}{2}[s_1+s_2\pm\sqrt{(s_1-s_2)^2+4\delta^2}]$. Thus, while the perturbed Hamiltonian (1) has nonvanishing off-diagonal blocks in the representation in which the symmetry wave functions are $\alpha_{1,2}$, it will recover the block-diagonal form when using the representation that involves the eigenvectors $\beta_{1,2}$ of S_δ,

$$H = \begin{pmatrix} H_1' & 0 \\ 0 & H_2' \end{pmatrix}_\delta . \tag{2}$$

Here, the diagonal blocks $H_{1,2}'$ are nearly equal to the diagonal blocks $H_{1,2}$ of the unperturbed Hamiltonian, except for a shift in the diagonal matrix elements by almost the same amount in each block, but in opposite directions. This shift will produce a corresponding shift in the eigenvalues corresponding to each block. If the level density of each sequence in the absence of perturbation is $\rho(E)$, then after perturbation, the level densities of the sequences become $\rho(E\pm a\varepsilon)$, respectively, where a is a constant. Now, if $\rho(E)$ is approximated by an exponential function as in many applications, then the fractional density of the sequence that decreases by the influence of perturbation, say the sequence labeled by 1, can be approximately related to the perturbation parameter ε by

$$f_1 = f_{1,0}e^{-A\varepsilon}, \tag{3}$$

where $f_{1,0}$ is the initial fractional density and A is a constant.

The spectrum of the chaotic system under consideration can approximately be represented by a superposition of two independent sequences all the way through the symmetry-breaking transition, even in the case when $H_1\neq H_2$ in Eq. (1). Analysis of the spacing distribution of coupled resonators of unequal size has shown this [10]. The purpose of the following sections is to provide further support for this approximation.

We note that the proposed representation of symmetry breaking is meant for chaotic systems. It does not apply, for example, to the experiment by Ellegaard *et al.* [14], where a gradual breaking of a point-group symmetry in monocrystalline quartz blocks is achieved by cutting small pieces at one of the angles. The properties of quartz allowed these authors to consider the pure sequences as pseudointegrable systems (see, e.g., Refs. [15–17]). A previous analysis [18] of this experiment has shown that the symmetry breaking can be explained by introducing a new GOE level sequence in addition to the original pseudointegrable ones, and allowing it to grow at their expense.

III. CALCULATION OF LEVEL-NUMBER VARIANCE

The level number variance for a spectrum composed of a superposition of M independent sequences is given by [19]

$$\Sigma_\beta^2(r,f_1,\ldots,f_M) = \sum_{m=1}^{M}\Sigma_\beta^2(f_m r), \tag{4}$$

where $\beta=1$, 2, or 4 depending on whether the pure sequences belong to GOE, GUE, or GSE, respectively, f_m is the average fractional density of the mth sequence, and $\Sigma_\beta^2(r)$ is the number variance for the pure sequence. A similar expression is proposed by Seligman and Verbaarschot [20] for the spectral rigidity Δ_3,

$$\Delta_{3\beta}(r,f_1,\ldots,f_M) = \sum_{m=1}^{M} \Delta_{3\beta}(f_m r). \quad (5)$$

French *et al.* [4] considered the gradual symmetry breaking by the influence of a perturbation represented by a random matrix with elements H_{ij}. They calculated the two-level correlation function using the perturbation theory. They expressed the level-number variance as

$$\Sigma^2_\beta(r,\Lambda) = \Sigma^2_\beta(r) + \frac{M-1}{\beta\pi^2}\ln\left[1 + \frac{\pi^2 r^2}{4(\tau_\beta + \pi^2\Lambda)^2}\right], \quad (6)$$

where τ_β is a cutoff parameter and Λ is the mean-square value of the perturbation measured in units of the mean level spacing D,

$$\Lambda = \overline{H^2_{ij}}/D^2 = \varepsilon^2 v^2/D^2. \quad (7)$$

The cutoff parameter is estimated by the requirement that, when $\Lambda=0$, $\Sigma^2_\beta(r,0)=\Sigma^2_\beta(r,f_1,\ldots,f_M)$ as given by Eq. (4). Imposing this requirement means the τ_β depends on the energy interval r. In previous application of this approach, e.g., in [6,9,10], an average over the range of r covered by the data is taken. We suggest here, instead, to determine the cutoff parameter by requiring the equality of Eqs. (4) and (6) at a large value of r, such that $r \gg 1/\min(f_m)$. For this purpose, we use the asymptotic expression for the number variance, which is given by

$$\Sigma^2_\beta(r) \sim \frac{2}{\beta\pi^2}\ln(2\pi r + c_\beta + \gamma + 1), \quad (8)$$

where $\gamma \cong 0.5772$ is Euler's constant and $c_\beta = -\pi^2/8$, 0, and $\ln 2 + \pi^2/8$ for GOE, GUE, and GSE, respectively. Combining Eqs. (4), (6), and (8), we obtain

$$\tau_\beta = C_M e^{-(c_\beta+\gamma+1)}, \quad (9)$$

where $C_M = \frac{1}{4}(\Pi_{m=1}^{M} f_m)^{1/(1-M)}$. The preexponential factor becomes $C_M = \frac{1}{4}M^{1/(1-1/M)}$ in the case when all the initial sequences have the same level density $f_m = 1/M$. In particular, if $M=2$, then $\tau_\beta = 0.709$, 0.207, and 0.082 for GOE, GUE, and GSE, respectively. In the case of two and eight GOE sequences of equal initial level density, Leitner [6] finds $\tau_1 = 0.70$ and 1.85, which are in good agreement with our result since Eq. (9) yields $\tau_1 = 0.709$ and 1.609 for $M=2$ and 8.

We now assume that the symmetry-breaking transition proceeds in such a way that one sequence, say with $m=I$, grows at the expense of the others. Then, $\Sigma^2_\beta(r,\Lambda)$ will again be expressed as a sum of contributions of the m sequences,

$$\Sigma^2_\beta(r,\Lambda) = \sum_{m=1}^{M} \Sigma^2_\beta[f_m(\Lambda) r], \quad (10)$$

where $f_m(\Lambda)$ is the average fractional density of the mth sequences in the presence of perturbation. We further assume, for a perturbation strength measured by Λ, the fractional level density of the sequences with $m \neq I$ decay with the same rate, so that

$$f_m(\Lambda) = f_m e^{-\xi_\beta(\Lambda)}, \quad (11)$$

where $\xi(\Lambda)$ is a monotonously increasing function of Λ. The condition that $\Sigma_{m=1}^{M} f_m(\Lambda) = 1$ yields

$$f_I(\Lambda) = 1 - (1 - f_I)e^{-\xi_\beta(\Lambda)}. \quad (12)$$

We can estimate the function $\xi_\beta(\Lambda)$ again by comparing Eqs. (4) and (6) in the asymptotic region of large r. In the case when all the initial sequences have equal level densities, $f_m = 1/M$, this comparison yields

$$\xi_\beta(\Lambda) = \pi\sqrt{\frac{2\Lambda}{M\tau_\beta}}, \quad (13)$$

which agrees with the estimate in Eq. (3), when $A = \pi v/D\sqrt{\tau_\beta}$. This finding shows the consistency of the argument leading to the model under consideration with the results of the perturbation theory concerning symmetry breaking. It also allows us to express the fractional level densities during the symmetry-breaking transition, $f_m(\Lambda)$, to the strength of the symmetry-breaking interaction (7).

IV. CALCULATION OF NNS DISTRIBUTION

The NNS distribution of a spectrum resulting from a random superposition of M independent sequences is well known [1], as mentioned above. Let $p_j(s)$ denote the NNS distribution for the jth subsequence. We now assume that each of the distributions $p_j(s)$ is that of a GOE. The generalization to other symmetry classes is straightforward. To an excellent approximation, the $p_j(s)$'s are then given by the Wigner surmise. We define the associated gap functions $E_j(s) = \int_s^\infty ds' \int_{s'}^\infty p_j(x)dx$ for the subsequences and the gap function $E(s) = \int_s^\infty ds' \int_{s'}^\infty p(x)dx$ for sequence S. Mehta [1] has shown that $E(s) = \Pi_{j=1}^{M} E_j(f_j s)$. The NNS distribution for the composite spectrum is obtained by differentiating $E(s)$ twice. When the symmetry has two eigenvalues, the spectrum is that of a superposition of two independent GOE level sequences of fractional densities f_1 and $f_2 = 1 - f_1$. The corresponding NNS distribution is given by

$$P(s,f_1) = \frac{\pi}{2} f_1^3 s e^{-\pi f_1^2 s^2/4} \mathrm{erfc}\left[\frac{\sqrt{\pi}}{2}(1-f_1)s\right] + \frac{\pi}{2}(1-f_1)^3 s e^{-\pi(1-f_1)^2 s^2/4}\left[\mathrm{erfc}\left(\frac{\sqrt{\pi}}{2} f_1 s\right)\right]^2 + 2f_1(1-f_1)e^{-\pi(2f_1^2-2f_1+1)s^2/4}. \quad (14)$$

The expression for the spacing distribution becomes more complicated when $M > 2$. We can obtain an approximate expression for the NNS distribution, valid for arbitrary M, that depends on a single parameter. This is the mean fractional level density $f = \Sigma_{i=1}^{M} f_i^2$ for the superimposed subsequence, where f_j are the fractional densities of the constituting ones. Some steps in this direction have already been previously taken [10,12,21,22].

The model proposed above, which represents the spectrum of a system with partial symmetry as a superposition of

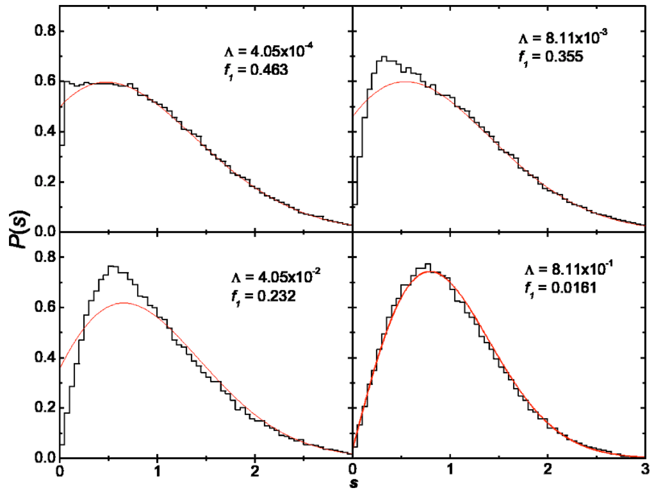

FIG. 1. NNS distributions during the 2GOE-GOE crossover transition. The histograms represent the numerical results of Leitner's [6] diagonalization of ensembles of two-block diagonal matrices, each of which is a GOE, perturbed by random real matrices with strength parameters $\Lambda = 4.05\times 10^{-4}$, 8.11×10^{-3}, 4.05×10^{-2}, and 8.11×10^{-1}. The smooth curves are for the prediction of the proposed independent-sequence model for symmetry breaking. The tuning parameter f_1 is calculated by using Eqs. (11) and (13) with the corresponding values of interaction strength Λ.

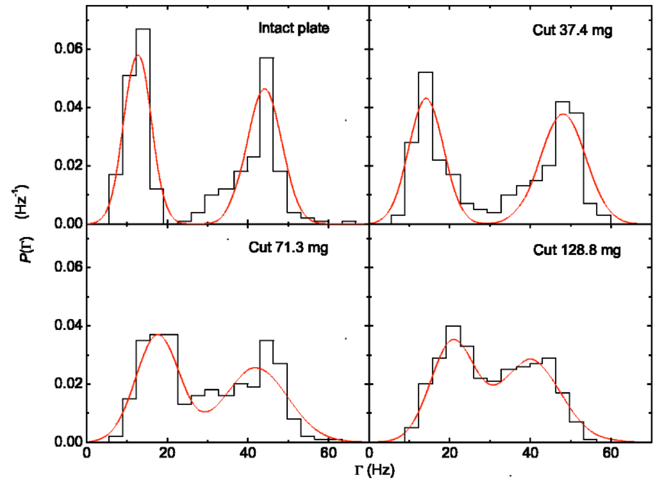

FIG. 2. Resonance-width distributions for the acoustic resonances in intact and cut three-leaf clover-shaped plates, measured by Anderson *et al.* [13], fitted by a sum of two Gaussian functions.

independent sequences, suggests applying Eq. (14) for the transition from the two-GOE statistics to that of a single GOE. The fractional density f_1 of the decaying sequence will then play the role of a tuning parameter. A weak point of the distribution in Eq. (14) is that it differs from zero at $s=0$, because the symmetry—breaking interaction lifts the degeneracies. The model thus fails in the domain of small spacings as far as the NNS distributions are concerned. The magnitude of this domain depends on the ratio of the strength of the symmetry—breaking interaction to the mean level spacing. However, this defect does not affect the long-range statistics (e.g., Σ^2 or Δ_3).

Equations (11) and (13) provide a relation between the fractional density of the decaying sequence and the symmetry—breaking strength. We now test this relation. Leitner [6] numerically diagonalized sets of real-symmetric matrices of the form in Eq. (1), where the matrices H_1, H_2 are independent GOE matrices of equal dimension and V contains interrelated Gaussian random variables. Each set corresponds to a different value of strength parameter ε [or Λ, given by Eq. (7)] and consists of about 2000 matrices of dimension 400. The NNS distribution obtained by Leitner for four values of $\Lambda = 4.05\times 10^{-4}, 8.11\times 10^{-3}, 4.05\times 10^{-2}$, and 8.11×10^{-1} is shown as histograms in Fig. 1. The figure compares these statistically significant numerical results with the prediction of Eq. (14), with f_1 calculated by inserting these values of Λ into Eqs. (11) and (13). We see that the proposed model presents a satisfactory agreement with the numerical results while using the same values for the parameter Λ. The only disagreement is between the calculated and measured values of $P(s)$ at small values of s, as we have already expected. The symmetry breaking decreases the probability of finding degenerate levels sharply, leading to the observed dip at small s in the spacing distributions of the numerical experiment. This dip is followed by an overshoot to restore normalization. The width of this dip, which is a measure of level splitting responsible for degeneracy removal, increases with increasing the parameter Λ, as expected. In spite of this, we shall find in the next section that the parameters obtained in the comparison of the NNS distribution (14) with experiment can successfully be used in the analysis of other statistics such as rigidity Δ_3 for the same spectra.

V. COMPARISON WITH EXPERIMENT

Andersen *et al.* [13] measured the frequency spectrum and the widths for acoustic resonances in thin aluminum plates, cut in the shape of a three-leaf clover with outer and inner radii 80 mm and 70 mm, respectively. This shape is chosen because a similar billiard has a chaotic classical dynamics [23]. Due to the mirror symmetry through the middle plane of the plate, each resonance of the plate belongs to one of two mode classes. The flexural modes, which have displacement mainly normal to the plane of the plate, are antisymmetric with respect to reflection through the middle of the plane. The in-plane modes are symmetric. The authors separated the modes according to their measured widths and showed that each mode class obeyed the GOE statistics. The number of observed modes in each class was nearly the same. They introduced a gradual breaking of the mirror symmetry by cutting a slit of increasing depth on one face of the plate. They were able to describe the transition that takes place as the mode classes mixed in terms of a random matrix model.

This section demonstrates that the resonances for the three-leaf clover plate can approximately be described as two uncoupled classes even if the symmetry is partially broken. Figure 2 shows by histograms the experimental width distribution for the resonances when the plate is intact, and when three different symmetry-breaking splits are cut out. The first case is naturally described as an independent superposition

TABLE I. Parameters used in the comparison of the width distribution with a superpesiton of two Gaussian functions, shown in Fig. 2.

	Intact plate	Cut 37.4 mg	Cut 71.3 mg	Cut 128.8 mg
C	0.5	0.47 ± 0.05	0.50 ± 0.05	0.47 ± 0.04
Γ_I	12.6 ± 0.3	14.2 ± 0.6	17.6 ± 0.7	20.7 ± 0.5
σ_I	2.8 ± 0.3	3.8 ± 0.6	5.0 ± 0.7	5.1 ± 0.5
Γ_F	44.1 ± 0.5	48.1 ± 0.8	42.0 ± 1.2	40.1 ± 0.7
σ_F	3.8 ± 0.4	5.3 ± 0.7	7.6 ± 1.2	7.2 ± 0.7

of contributions from the in-plane and flexural modes. The authors of [13] assume that the width distribution in this case is given by a linear superposition of two Gaussian distributions of the same weight, i.e.,

$$P(\Gamma) = Cg(\Gamma_I^0,\sigma_I^0) + (1-C)g(\Gamma_F^0,\sigma_F^0), \tag{15}$$

where $g(\Gamma,\sigma)$ is a Gaussian distribution with mean value Γ and variance σ^2, and the suffixes I and F are for the in-plane and flexural modes, respectively. The statistical weights of both classes are set equal, i.e., $C=\frac{1}{2}$. They analyzed the width spectrum of the distorted plates using a random matrix model that describes the coupling of the two mode classes. Figure 2 analyzes the width distribution for both the intact and distorted plates by the sum of two Gaussians in Eq. (15). The statistical weights, mean widths, and variances for each mode are determined by a χ^2 fit. The figure shows that the overlap of the width distributions of the two classes indeed increases with symmetry violation. However, the resolution of the two modes by means of Eq. (15) is possible even in the case where the symmetry is almost completely destroyed. Interestingly, the statistical weights of the two modes in all cases remain the same, equal to $\frac{1}{2}$, within the statistical error.

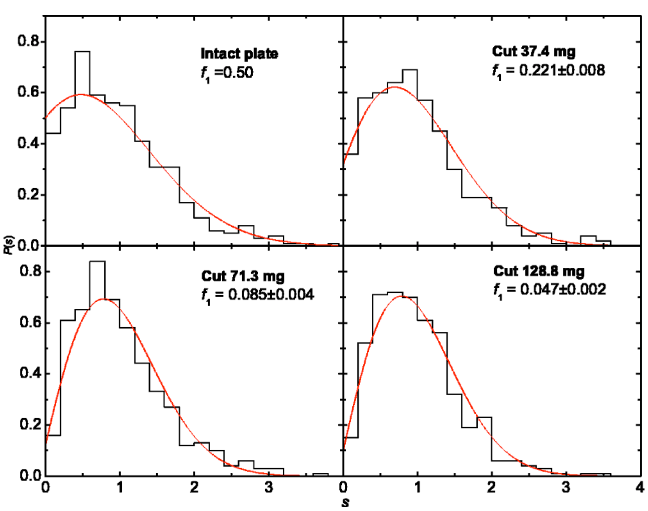

FIG. 3. NNS distributions for the acoustic resonances in intact and cut three-leaf clover-shaped plates measured by Anderson *et al.* [13]. The curves are the results of χ^2 fits by Eq. (14) for a spectrum composed of two independent GOE sequences.

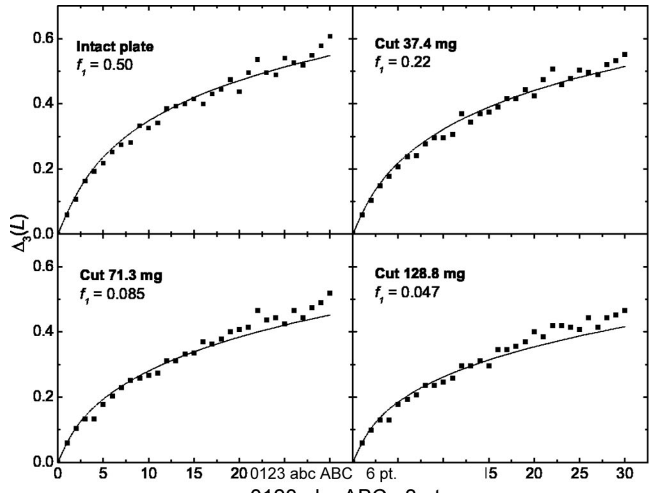

FIG. 4. Spectral rigidity Δ_3 for the acoustic resonances in intact and cut three-leaf clover-shaped plates measured by Anderson *et al.* [13]. The curves are calculated for a superposition of two independent GOE sequences with the same fractional densities that fit the corresponding NNS distribution in Fig. 3.

Figure 3 compares the resonance spacing distributions measured in [13] with the prediction of Eq. (14). The fractional density of the decreasing sequence f_1 is set equal to 0.5 in the case of an impact plate, where the symmetry is conserved, and considered as a fitting parameter in the cases of deformed plates. The best-fit values are 0.221 ± 0.008, 0.085 ± 0.004, and 0.047 ± 0.002. The agreement with the experiment is good. An exception is the domain of small spacings (the first bin), where the model predicts nonvanishing distribution at $s=0$.

Figure 4 compares the experimental values of the Δ_3 statistic for the same four cases considered in Fig. 3 with the ones calculated for a corresponding superposition of two sequences. The fractional densities of the sequences used in the calculations are the best-fit values obtained for the NNS distributions. The agreement between the theoretical curves and experimental histograms is good. This good agreement means that the evaluation of the fractional densities of the sequences constituting the spectrum using Eq. (14) for the NNS distribution is accurate despite the wrong behavior of this distribution at small spacings.

VI. CONCLUSION

We propose a model for the spectral fluctuations of systems with partially conserved symmetry. When the symmetry is exact, the spectrum is composed of a superposition of independent level sequences, each corresponding to a fixed value of the symmetry quantum number. We argue that the same representation may still be valid when the symmetry is violated. The symmetry-breaking transition is modeled by assuming that one of the sequences is growing at the expense of the others. A relation between the fractional level densities of the intermediate sequences and the symmetry-breaking interaction strength is obtained by comparing the asymptotic behavior of the level-number variance for the sequence superposition with the previous results obtained by applying the perturbation theory. The model is tested by comparing its prediction with the results of numerical diagonalization of a Hamiltonian divided into a symmetry-conserving term and a perturbation and the outcome of an acoustic resonance experiment. It is found to give an accurate representation for the spectra except in the domain of small spacing, where the symmetry-breaking interaction removes all possible accidental degeneracies.

The proposed model is not meant to replace more sophisticated models that describe the breaking of known symmetries, such as isospin or parity. However, it is useful in cases when an approximate symmetry is unknown or ignored. It has recently been successfully applied to study the NNS distribution of low-lying 2^+ states of even-even nuclei. Relatively small values for the mean fractional level densities of the superimposed sequences are obtained for nuclei expected to have one of the dynamical symmetries of the interacting boson model, indicating that their spectra may be divided into two or more nearly independent sequences.

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